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### Path regularity and explicit convergence rate for BSDE with truncated quadratic growth

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Corrigendum to “Path regularity and explicit convergence rate  
for BSDE with truncated quadratic growth” [Stochastic Process.  
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This short note is written to replace the defective proof of Theorem 5.5-(ii) in [1]. The result remains true without any additional assumptions. For this reason we literally adopt all the notations, assumptions and equation numbers used in [1]. The error in the mentioned proof originates in a misapplication of Hölder’s inequality in the estimate of  $I_1$  (see page 370 of [1] two lines after (29)).

**Theorem 5.5 Part (ii) in [1]:** Under HX1 and HY1, the FBSDE system (1), (2) has a unique solution  $(X, Y, Z) \in \mathcal{S}^{2p} \times \mathcal{S}^\infty \times \mathcal{H}^{2p}$  for all  $p \geq 1$ . Moreover, the following holds true:

- (ii) For all  $p \geq 1$  there exists a constant  $C_p > 0$  such that for any partition  $\pi$  of  $[0, T]$  with  $N$  points and mesh size  $|\pi|$

$$\sum_{i=0}^{N-1} \mathbb{E} \left[ \left( \int_{t_i}^{t_{i+1}} |Z_t - Z_{t_i}|^2 dt \right)^p \right] \leq C_p |\pi|^p.$$

*Proof.* Throughout fix  $p \geq 1$ . Theorem 5.3 states that  $Z \in \mathcal{S}^{2p}$  and therefore, using Jensen’s inequality and Fubini’s theorem we are able to write

$$\mathbb{E} \left[ \left( \int_{t_i}^{t_{i+1}} |Z_t - Z_{t_i}|^2 dt \right)^p \right] \leq |\pi|^{p-1} \int_{t_i}^{t_{i+1}} \mathbb{E} [ |Z_t - Z_{t_i}|^{2p} ] dt.$$

Using Theorem 5.2 and the representation formulas of Theorem 2.9 we can rewrite the difference inside the expectation as  $Z_t - Z_{t_i} = J_1 + J_2 + J_3$  with  $J_1 = [\nabla Y_t - \nabla Y_{t_i}](\nabla X_{t_i})^{-1} \sigma(X_{t_i})$ ,  $J_2 = \nabla Y_t [(\nabla X_t)^{-1} - (\nabla X_{t_i})^{-1}] \sigma(X_{t_i})$  and  $J_3 = \nabla Y_t (\nabla X_t)^{-1} [\sigma(X_t) - \sigma(X_{t_i})]$  (with  $t \in [t_i, t_{i+1}]$ ).

Estimates for  $J_2$  and  $J_3$  are easy to obtain since they rely mainly on the fact that  $\nabla Y \in \mathcal{S}^q$  for all  $q \geq 2$  and the known estimates for SDEs found for instance in Section 2.5. We give details for  $J_2$  and hints on how to deal with  $J_3$ , remarking that its treatment is very similar.

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Hölder's inequality combined with the growth condition of  $\sigma$  produce for  $t \in [t_i, t_{i+1}]$

$$\mathbb{E}[|J_2|^{2p}] \leq C \|\nabla Y\|_{\mathcal{S}^{6p}}^{2p} \mathbb{E}\left[|(\nabla X_t)^{-1} - (\nabla X_{t_i})^{-1}|^{6p}\right]^{\frac{1}{3}} (1 + \|X\|_{\mathcal{S}^{6p}}^{2p}) \leq C |\pi|^{3p\frac{1}{3}} = C |\pi|^p.$$

Where in the last line we used (4), (8) and  $\|\nabla Y\|_{\mathcal{S}^q} < \infty$  for any  $q \geq 2$ . For  $J_3$ , the method is similar: instead of (4) and (8) one uses (5) and (7) combined with HX0.

At this point it is fairly easy to see that

$$\sum_{i=0}^{N-1} |\pi|^{p-1} \int_{t_i}^{t_{i+1}} \mathbb{E}[|J_2|^{2p} + |J_3|^{2p}] ds \leq \sum_{i=0}^{N-1} |\pi|^{p-1} (t_{i+1} - t_i) C |\pi|^p = C T |\pi|^{2p-1}.$$

To handle the term  $J_1$  one needs to proceed with more care. Let us start with a simple trick:

$$\mathbb{E}\left[|(\nabla Y_t - \nabla Y_{t_i})(\nabla X_{t_i})^{-1}\sigma(X_{t_i})|^{2p}\right] = \mathbb{E}\left[\mathbb{E}[|\nabla Y_t - \nabla Y_{t_i}|^{2p} | \mathcal{F}_{t_i}]\right] |(\nabla X_{t_i})^{-1}\sigma(X_{t_i})|^{2p}.$$

Writing the BSDE for the difference  $\nabla Y_t - \nabla Y_{t_i}$  for  $t_i \leq t \leq t_{i+1}$  we have for some positive constant  $C$

$$\begin{aligned} \mathbb{E}\left[|\nabla Y_t - \nabla Y_{t_i}|^{2p} | \mathcal{F}_{t_i}\right] &\leq C \mathbb{E}\left[\left|\int_{t_i}^t \langle (\nabla f)(r, \Theta(r)), (\nabla \Theta)(r) \rangle dr\right|^{2p} + \left|\int_{t_i}^t \nabla Z_r dW_r\right|^{2p} | \mathcal{F}_{t_i}\right] \\ &\leq C \mathbb{E}\left[\left(\int_{t_i}^{t_{i+1}} |(\nabla f)(r, \Theta_r)| |\nabla \Theta_r| dr\right)^{2p} + \left(\int_{t_i}^{t_{i+1}} |\nabla Z_r|^2 dr\right)^p | \mathcal{F}_{t_i}\right]. \end{aligned}$$

Here we used the conditional Burkholder-Davis-Gundy inequality and maximized over the time interval  $[t_i, t_{i+1}]$ . For convenience of notation we define the sum of the integrals inside the conditional expectation by  $\widehat{J}_{[t_i, t_{i+1}]}$ .

Combining these last two inequalities and observing that since  $\nabla X_{t_i}$  and  $\sigma(X_{t_i})$  are  $\mathcal{F}_{t_i}$ -adapted we can drop the conditional expectation. This way for some positive constant  $C$  we obtain

$$\begin{aligned} |\pi|^{p-1} \sum_{i=0}^{N-1} \int_{t_i}^{t_{i+1}} \mathbb{E}\left[\mathbb{E}[|\nabla Y_t - \nabla Y_{t_i}|^{2p} | \mathcal{F}_{t_i}]\right] |(\nabla X_{t_i})^{-1}\sigma(X_{t_i})|^{2p} dt \\ \leq C |\pi|^{p-1} \sum_{i=0}^{N-1} |\pi| \mathbb{E}\left[\widehat{J}_{[t_i, t_{i+1}]} |(\nabla X_{t_i})^{-1}\sigma(X_{t_i})|^{2p}\right] \\ \leq C |\pi|^p \mathbb{E}\left[\sup_{0 \leq t \leq T} |(\nabla X_t)^{-1}\sigma(X_t)|^{2p} \sum_{i=0}^{N-1} \widehat{J}_{[t_i, t_{i+1}]}\right] \\ \leq C |\pi|^p \mathbb{E}\left[\sup_{0 \leq t \leq T} |(\nabla X_t)^{-1}\sigma(X_t)|^{2p} \left\{\left(\int_0^T |(\nabla f)(r, \Theta_r)| |\nabla \Theta_r| dr\right)^{2p} + \left(\int_0^T |\nabla Z_r|^2 dr\right)^p\right\}\right] \\ \leq C |\pi|^p. \end{aligned}$$

The last line follows from a combination of inequality (25), assumption HY1 (namely the growth conditions for the derivatives of  $f$ ) and the fact that for any  $q \geq 2$  we have:  $X, \nabla X, (\nabla X)^{-1} \in \mathcal{S}^q$ ,  $Y, Z, \nabla Y \in \mathcal{S}^q \cap \mathcal{H}^q$  and  $\nabla Z \in \mathcal{H}^q$ .

Collecting now the estimates on  $J_1$ ,  $J_2$  and  $J_3$  we obtain the desired result.  $\square$

## References

- [1] P. Imkeller and G. Dos Reis. Path regularity and explicit convergence rate for BSDE with truncated quadratic growth. *Stochastic Process. Appl.*, 120(3):348–379, 2010. ISSN 0304-4149. doi: 10.1016/j.spa.2009.11.004.